

# Modeling of Time and Distance Decay Functions for Modes of Transportation in the City of San Juan de Pasto - Nariño, Colombia

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## Abstract

The role of urban transport and its accessibility to key destinations are fundamental in urban planning and as part of the quality of life. This study focuses on the city of San Juan de Pasto, Colombia, aims to estimate the decay functions of time and distance for four modes of transportation: Bicycle, private vehicle, public transportation, and walking. The decay curves were modeled as exponential and Gaussian functions using empirical data collected from local mobility surveys. The findings indicate substantial distinctions in residents' propensity to make trips given the length and duration of trips, as well as the form of transportation.

Mean distances and shorter walking and cycling time were observed compared to private vehicles and public transport for longer trips. The results show that exponential functions better model travel behaviors on foot and in particular vehicles since they tend to detach from the access table with distance and time. In contrast, Gaussian functions were better suited to public transport and bicycles by capturing a gradual decrease in accessibility over greater distances. On the walking trip, they averaged 1 km or 12 minutes, and about 5 km or 20 minutes on the bike trip. It is therefore a matter of journeys of up to 15 km, that is, 25 minutes by private car or up to 10 km, that is, approximately 40 minutes by more public transport. Finally, an analysis of the decay functions of the travel series is presented, and it is shown that the exponential function optimally represents the data series, and that it is useful in particular for modes of transport that tend not to be motorized (walking and cycling) due to the advantages of its dispersion and better statistics. As required by the high variability of times and distances, motorized modes, such as the private vehicle, demand greater modeling efforts, for example, with the Gaussian function. Analysis of the beta parameters of decay functions provides us with additional information about transport patterns. The results highlight the importance of including the characteristics of each mode in the design of mobility policy, especially the infrastructure that encourages short-distance travel in active modes and that makes it possible to travel long distances in motorized modes. The opportunity to approach maximizes the efficiency of the system and the sustainability of urban transit systems

**Keywords:** Urban transport; accessibility; the functions that absorb the load; modes of transport; urban planning.

## 1. Introduction.

Decay functions are mathematical models for determining the probability of a given individual making a trip by using an associated cost in the form of space, time, or effort. These functions are crucial to make people understand the obstacles that users must deal with in order to design transport systems more equitably and efficiently. Among these, exponential, Gaussian and potential theories stand out, each adapted to different contexts and purposes of analysis (Handy & Niemeier, 1997).

The importance of decay functions is based on the fact that they are used for the calculation of indicators of potential accessibility, opportunities or severity focus. A set of these indicators that combines land use information along with transportation allows us to assess how opportunities for interaction deteriorate as distance or travel time increases. Cabrera-Barona, Blaschke and Kienberger (2017) have used these indicators to detect spatial imbalances in access to essential services, highlighting their usefulness in urban planning.

Numerous studies, on a global regional scale, have dealt with spatial and urban accessibility. Hansen (1959) introduces the concept of accessibility as a potential for opportunities and interaction has been refined since then through different approaches. For example, Geurs and Van Wee (2004) propose that accessibility be assessed based on four main components: it includes land use, transportation, time, and individual characteristics. All these dimensions make it possible to talk about accessibility from both individual and collective perspectives, in order to bring together the needs of urban and rural populations.

Research such as that of Martínez and Viegas (2013) in Bogotá shows that for Latin America, accessibility would not depend only on transport infrastructure, but also on the socioeconomic and geopolitical barriers faced by users. On a global level, the work of Iacono (2008) has highlighted that decay functions are useful in the modeling of travel behavior, providing tools for a better understanding of the dependence between travel costs and mobility patterns.

Understanding and analysing transport systems is essential for the development of sustainable and equitable cities, and in the case of urban contexts it is crucial to understand and improve their function, as mobility and accessibility are determinants of the degree of quality of life. One of them is evaluated from different perspectives, such as traffic, mobility and accessibility. In particular, accessibility has emerged as a key indicator because it is the degree of ease with which people can achieve activities, services, and opportunities in a given geographical space (Geurs and Van Wee, 2004). This perspective keeps transport planning very closely linked with land use, seeking solutions that integrate infrastructure, services and spatial equity.

Some studies, such as those by Litman (2003) and Hu et al. (2013), highlight how such tools can be applied to the formulation of sustainable mobility policies. For example, decay functions have been used in Bogotá to model the accessibility of the TransMilenio mass transit system, finding areas with low connectivity and proposing specific interventions for access equity.

This study contributes to this line of research, focusing on the intermediate city of San Juan de Pasto, capital of the department of Nariño in Colombia. The particular topography of this city and its compact urban configuration constitute a unique environment in which to analyse how physical and socio-economic barriers affect the mobility decisions of its inhabitants. This work will provide useful tools for the planning and design of urban policies that promote sustainability and equity of access to opportunities by estimating the decay functions of time and distance for four modes of transport (walking, cycling, car, mechanical traction).

## 2. Theoretical Framework

Accessibility, a multidimensional concept widely studied in the transport and urban planning literature, has been defined from various perspectives that reflect its complexity and scope. According to Geurs & Van Wee (2004), multiple approaches have been established that conceptualize accessibility, among others, such as the ability of individuals to spatially obtain opportunities through a transport system (Dalvi and Martin 1976), the existence of potential interaction with different land uses (Hansen 1959) or even the economic benefits derived from participation in activities facilitated by transport infrastructure (Ben-Akiva & Lerman, 1977). These approaches exhibit the dynamic relationship between transport systems, their organization of the territory and the peculiarities of the individual population. Accessibility is structured around four essential components: In terms of land use, transport, temporality and the individual characteristics of users. While land use accounts for the spatial distribution of opportunities in terms of quantity, qualities and validity between supply and demand at a given time, transport proposes functional connectivity between the nodes of the system, for example, in terms of the distance for their movement, the time of travel, cost and effort required in development (Litman, 2020). At the same time, the temporal component implies temporal constraints that have to do with the availability of

opportunities as a function of time, and individuals reflect asymmetries in accessibility that derive from socioeconomic factors, physical abilities, and availability of appropriate means of transportation (Papa et al., 2016). To assess accessibility, various methodologies have been developed based on these components, grouped into four main approaches: These include infrastructure-based measures that analyse the capacity of the transport system; location-based measures that describe spatial accessibility to distributed opportunities; measures focused on individuals that consider personal restrictions and preferences; and care-oriented measures that address accessibility from a cost-benefit perspective (Geurs & Van Wee, 2004). In this context, decay functions are fundamental in the modelling of accessibility, since they allow quantifying the reduction in perceived usefulness of a destination as spatial friction increases, properly expressed from the distance or travel time. Exponential, logistic and power decay models are used to characterize the different patterns of accessibility depending on the mode of transport and the spatial configuration of opportunities, as fundamental tools for decision-making at the level of transport planning and evaluation of urban accessibility policies. The incorporation of these models into empirical analyses makes it possible to design mobility strategies based on equity and efficiency, as well as sustainable and inclusive urban principles.

Decay functions are mathematical representations of how the probability of making a trip decreases the higher the associated cost, either in time or distance. In these functions, the underlying principle is that as the generalized cost (indicated as C) rises, the probability that a user will decide to make the trip decreases. The behavior involved with users' perception of disutility. The mathematical probability of making that trip is representable in terms of a decreasing function with the associated cost. The most commonly used are exponential and Gaussian functions that are calibrated with aggregate data regressions (Handy & Niemeier, 1997). Decay functions describe the drop over time in the probability of access or perceived usefulness of a destination as travel time or distance to travel increases. They are used in accessibility analyses to measure how the spatial distribution of services and opportunities are being affected by the mobility condition. Mathematically, they can be represented by:

- **Exponential function:**

$$f(C) = e^{-\beta d}$$

Where C represents the generalized cost (money, distance, or time), and  $\beta$  is the decay rate, indicating how quickly accessibility is reduced with the assessed cost such as distance or other.

- **Logistics function:**

$$f(t) = \frac{1}{1 + e^{\beta(t-t_0)}}$$

This feature is used to capture thresholds of travel resistance, where the perception of accessibility drops steeply after a certain time  $t_0$ .

- **Potential Function:**

$$f(C) = d^{-\beta}$$

Suitable for systems where travel friction is lower and attraction patterns are observed over long distances. C is the distance

- **Gaussian function**

$$f(C) = e^{-\left(\frac{C-\mu}{\sigma}\right)^2}$$

This function is suitable for representing cases where decay is not linear, and cost extremes have less impact.

Where:

- $\mu$ : Mean or central value around which the probability of making a trip is concentrated.
- $\sigma$ : Standard deviation that defines the dispersion of values around  $\mu$ .

Accessibility has been introduced by Hansen (1959), a concept that has then evolved into the integration of components such as transport and land use (Geurs and Van Wee, 2004) as the potential for interaction opportunities. In this sense, decay functions are of vital importance for the calculation of potential accessibility indicators that are combined with mobility and spatial distribution information to analyze inequalities in access to essential services (Cabrera-Barona et al., 2017).

The application of least squares regressions to fit these empirical data to decay functions is part of the modeling of these curves. Beta coefficients obtain not only the forms of transport in which users are willing to travel, but also the properties of the transport network and its efficiency. Exponential functions were better suited in non-motorized modes in Pasto, and Gaussian functions better captured preferences in motorized modes.

People's travel behavior results from the interaction between the transport supply with the characteristics of land use and individual needs. In the situation of Pasto, this behavior is studied based on the 2017 Mobility Survey, which reports a more detailed overview of mobility patterns in the city. Key information from these surveys on distances traveled, travel times, and preferred modes of transportation allowed modeling the decay function. Through this survey, it was possible to establish that pedestrians make shorter trips in time and distance than users of public transport, who are also willing to take longer trips. For example, the results show that pedestrians usually walk on average for 1 km or 12 minutes while public transport users usually travel a distance of up to 10 km or 40 minutes on average. Given these are necessary to model decay curves and to plan more equitable transport systems. In the case of Pasto, these functions allow the calculation of the Beta parameters, which describe the sensitivity of users to travel costs in four main modes of transport: pedestrian, bicycle, private vehicles and public transport.

Location-based measures of accessibility are basically a fundamental category in spatial analysis and in transportation, so they are desired to test the degree of access of a geographical site against the distribution of opportunities surrounding the site. These can be classified into several types among which are contained: distance and contour measurements, potential measurements, and balance factors of spatial interaction models. Measures of potential accessibility, particularly so-called measures of gravity, have been widely used in urban and geographic studies because of their ability to calculate the accessibilities of opportunities offered in one proposed area versus another in the territory. All these indicators are based on the assumption that accessibility is inversely proportional to the distance or travel time required to access the opportunities offered. Mathematically, potential accessibility is represented by the following expression, under the assumption of a negative exponential cost function (Iacono, Krizek, & El-Geneidy, 2008; Harris, 2001):

$$A_i = \sum_{j=1}^n D_j e^{-\beta c_{ij}}$$

Where  $A_i$  represents accessibility in zone  $i$  for all opportunities  $D$  in zone  $j$ ;  $C_{ij}$  is the cost of travel between zones  $I$  and  $J$ , while it is the decay function that incorporates a parameter of sensitivity to the cost  $\beta$ .  $e^{-\beta c_{ij}}$

This parameter determines the degree of impedance of the trip, reflecting the decrease in interaction between two zones as their spatial separation increases. When  $\beta$  is relatively small, the impedance is low, which means higher accessibility; on the contrary, high  $\beta$  values indicate greater spatial friction and, therefore, less accessibility (Iacono, Krizek, & El-Geneidy, 2008). These measures are particularly useful for assessing spatial connectivity patterns in urban and peri-urban areas, providing a solid basis for the design of transport and territorial planning policies aimed at optimising access to economic and social opportunities.

The most frequently used decay functions are exponential, potential, logarithmic and Gaussian functions, which are closely linked to the theory of travel behavior (Handy and Niemeier, 1997), Table 1 presents the mathematical expressions of some of these functions

Function	Characteristics	Typical Applications	Expression
Exponential	Simplicity, uniform decrease	Travel allocation models and flow analysis.	$f(C_{ij}) = e^{-\beta(C_{ij})} \beta > 0$
Gaussian	Greater flexibility, represents focused distributions	Targeted cost assessments and localized analysis	$f(C_{ij}) = C_{ij}^{-\beta}, \beta > 0$

Where  $\beta$  is the constant to be calibrated and  $C_{ij}$  is the value of the impedance either in terms of distance, time, or cost of travel between zones  $i$  and  $j$ .

### 3. Methodology

3.1. Study Design: In the present work, time and distance decay functions were modeled for the modes of transport in the city of San Juan de Pasto, Colombia. The focus of the study was quantitative, applying empirical data collected through the use of mobility surveys to a representative sample of households. Here are some of these surveys, conducted in 2017, which provided important inputs of data on mobility patterns, such as distances and travel times, modes of transport, etc.

3.2. Data collection. The data comes from the "2017 Mobility Survey" carried out by Avante SETP and Movilidad Sostenible Ltda., under the umbrella of the Master Plan for Mobility and Public Space of the municipality of Pasto. The surveys were carried out between September 19 and October 20 and captured information on citizens' trips to attend work and/or study places. This input made it possible to build a robust database to analyze accessibility as a function of time and distance in four modes of transport: walking, cycling, private vehicle and public transport.

#### 3.3. Data Processing.

1. Data Transformation: Time data was converted into distances using average speeds adjusted for each mode of transportation and cumulative frequencies for trips were calculated as a function of time and distance.

2. Model Fit: Time and distance curves were modeled through exponential decay and Gaussian functions. The functions were then adjusted using least-squares regressions.

3. Model Validation: As in the tests, statistical metrics such as the coefficient of determination ( $R^2$ ), the mean square error (MSE) and the root mean square error (RMSE) were used to evaluate its accuracy. The values of the  $\beta$  parameter for each of the modes of transport were calculated, collecting the sensitivity of users to the costs of time and distance.

3.4. Data Analysis: We analyze the cumulative absolute frequencies of trips by time and distance for each mode of transport using graphical tools. Finally, the gravity-based accessibility measurements obtained were then integrated with  $\beta$  obtained for spatial friction assessment that are perceived by users.

#### 3.5. Modeled Results: Graphical representations were generated containing:

1. Cumulative absolute frequencies of trips according to time and distance.

2. Curve-adjusted decays for each mode of transportation.

3. Comparison between exponential and Gaussian functions, highlighting the suitability of each model according to the modality and the variable analyzed

## 4. Results

### 4.1. Source and description of the data

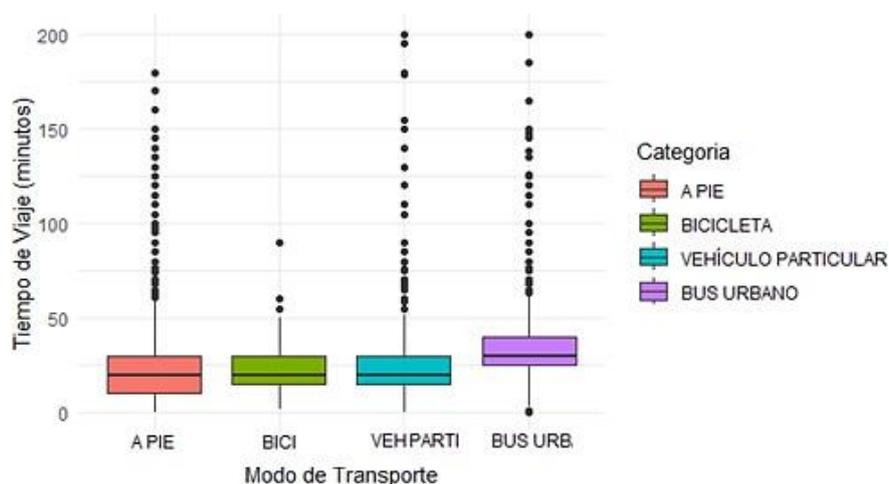
The study was based on data collected from a mobility survey of a representative sample of households retained in Pasto. In 2017, as part of the Pasto Mobility and Public Space Master Plan, the company Avante SETP, in partnership with the consulting firm Movilidad Sostenible Ltda., carried out a series of household surveys in different homes in that city. The main surveys of these were to collect general data about the housing and modes of transportation of each household of its members, including their trips to work or study sites.

The surveys were conducted between September 19 and October 20, 2017, from 4:00 p.m. to 9:00 p.m., Tuesday through Saturday. The information obtained from these surveys served as a fundamental input for the preparation of the technical document that guides the sustainable development of mobility in Pasto over the next 15 years. This plan seeks to establish policies, projects and strategies that efficiently integrate mobility, means of transportation, public space, territorial planning and human development in the municipality.

The data included information on the modes of transportation used, travel distances and times, as well as the demographic characteristics of the respondents.

### 4.2. Results

Figure 1 shows the distribution of travel times that people are willing to travel in the four modes of transport evaluated. The results indicate significant differences in decay functions for the four modes of transport

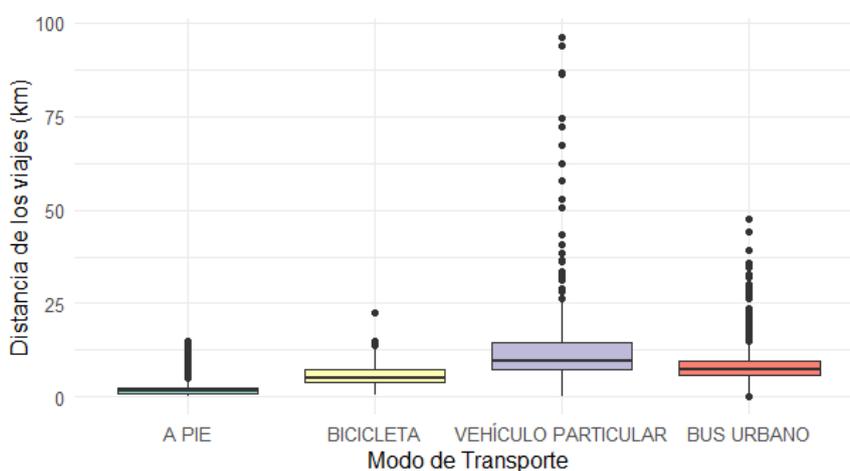


In original language Spanish

Figure 1. Travel times for people in the four modes of transport.  
Source: present research

In the walking mode, it has travel times that people take to travel in a certain mode of transport with a median of close to 20 minutes, with an interquartile range (IQR) that varies approximately between 10 and 40 minutes. This indicates that, for most users, walking is a viable option for short and moderate journeys. However, outliers are observed that reach up to 200 minutes, which may be associated with users who travel long distances or lack access to other means of transport. The

lower dispersion in the data reflects the stability of this mode in predictable paths and geographical proximity. As for the bicycle, it stands out as the most efficient and consistent mode of transport in terms of travel time. The median is around 15 minutes, with an IQR ranging from 10 to 30 minutes, making it a preferred choice for short and medium commutes. Outliers are less frequent and do not exceed 90 minutes, suggesting that this medium is reliable even for longer journeys. Low dispersion means adaptability and efficiency that may be related to the lack of traffic congestion and direct route opportunities. Travel time in a private vehicle with a median of 20 minutes (IQR 15–40), times similar to walking transport that have more variation. Outliers or outliers that exceed 150 minutes are observed, probably due to factors such as rush hour traffic, road accidents or long distances. Although it is a means that allows greater comfort and speed on average, the high dispersion in the data reveals limitations in the reliability of travel time, mainly influenced by urban mobility conditions. Urban bus transport has the highest and most dispersed travel times. The median reaches 30 minutes, while the IQR extends from 20 to 60 minutes, reflecting that most users face significantly longer times compared to other modes. In addition, outliers are numerous, with times exceeding 200 minutes, likely attributable to delays, road congestion and long distances. These results indicate that, although the bus is an accessible and popular means of transport, its efficiency in terms of time is limited, especially in contexts of high urban density. Figure 2 shows the distribution of travel distances in kilometers that people are willing to travel in the four modes of transport evaluated



In original language Spanish

Figure 2. Travel distance for people in the four modes of transport.

Source: present research

This figure presents the distribution of the distances that people are willing to travel in the different modes of transport, where it is observed that: pedestrians have a median of approximately 1 km, with an interquartile range (IQR) that extends between 0.5 km and 2 km, which reflects that this mode is mainly used for short distances. Although there are outliers that reach 10 km, these are uncommon and represent a significant physical effort for users. This result underlines the role of walking transport as a means limited to local routes, highly dependent on the proximity of destinations. On the bicycle, the median of 2 km and the IQR between 1 km and 5 km represent the preferred use of medium-distance routes. Outliers extend up to 25 km, indicating that a minority use this means rather than as a vehicle for short-distance trips. The low dispersion for a relatively wide range within the interquartile range reflects the elasticity of this medium and the reduced reliance on congested road infrastructure for short and medium distances. The travel distances of the private vehicle show the greatest dispersion, with a median close to 10 km (often short neighborhood trips) and an IQR between approximately 5 and 25 km. Outliers do not exceed 100 km, suggesting that this vehicle dominates longer journeys. This is the most versatile mode in terms of distance, reflecting the versatility of private vehicles in terms of adaptation to both urban and interurban contexts. However, it does so with high dispersion, its dependence on traffic and geographical characteristics. The city bus registers a median of 5 km, with an IQR between 3 km and 10 km, positioning it as a suitable mode for intermediate distances. Outliers reach up to 50 km, reflecting their use on longer routes, possibly to peripheral or rural areas. Although

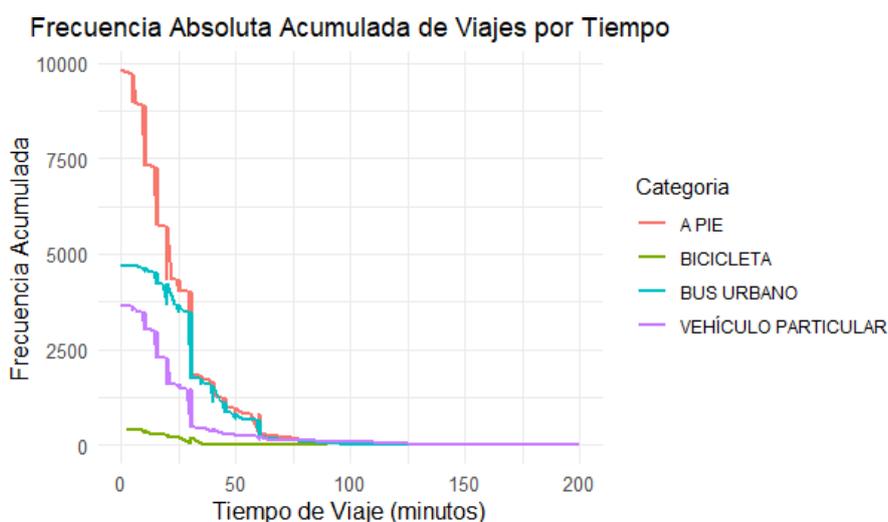
the dispersion is less than that of private vehicles, this result highlights the role of public transport in connecting different regions of the city, although it may be limited by infrastructure and operating frequencies.

### Construction of the time and distance decay function

Based on the information collected on travel times and distances for the four modes of transport analyzed, the absolute cumulative frequency of trips was determined, ordering them from highest to lowest in terms of time and distance, as shown in Figures 3 to 6. Subsequently, distance decay curves were fitted using exponential regression functions and Gaussian functions, described above. The adjustment was made using a least-squares regression analysis, which allowed estimating the value of the  $\beta$  parameter associated with each mode of transport (see Figures 5 and 6).

To ensure the selection of the most appropriate model, statistical indicators such as the coefficient of determination ( $R^2$ ), the mean square error (MSE) and the root mean square error (RMSE) were calculated. These metrics made it possible to identify the curve that best fit the observed data. The  $\beta$  parameter obtained from the selected model was subsequently employed in the calculation of the severity-based accessibility measures, as this parameter captures the sensitivity of the cost in terms of time and distance.

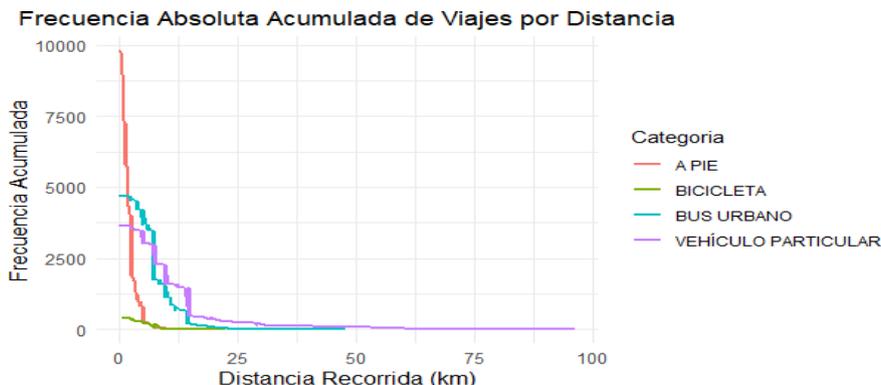
Finally, the graphs that summarize the results obtained are presented: cumulative absolute frequency of trips by time and distance, absolute frequency of trips according to time and distance, curves adjusted by exponential and Gaussian functions, as well as the specific adjustments for each mode of transport (pedestrian, bicycle, private vehicle and urban bus). These graphical representations allow a clear visualization of the behavior of the data and the adjustment of the selected models.



In original language Spanish  
Figure 3. Cumulative absolute frequency of trips based on travel time  
Source: present research

The graph represents the absolute cumulative frequency of trips as a function of travel time (in minutes) for four categories of transport: walking, cycling, city bus and private vehicle. Walking mode (red line) shows the highest accumulation of trips with travel times of less than 20 minutes, reaching a maximum value of 10,000 cumulative trips in a span of approximately 50 minutes. This pattern reflects their preference for short journeys due to physical limitations inherent in the mode of transport. In contrast, the bicycle (green line) has a significantly lower accumulation, with a maximum frequency of close to 1,000 trips, distributed mainly between 10 and 30 minutes, which suggests its use on moderate journeys. The urban bus (light blue line) and the private vehicle (purple line) exhibit more dispersed behaviors. In the case of the urban bus, the accumulation reaches a maximum of 4,000 trips in travel times of up to 60 minutes, showing a wide range of use for intermediate and long distances. On the other hand, the private vehicle has a lower cumulative number of

trips (2,500), distributed more evenly in times of up to 100 minutes, with less slope, indicating its use for both urban and interurban journeys.

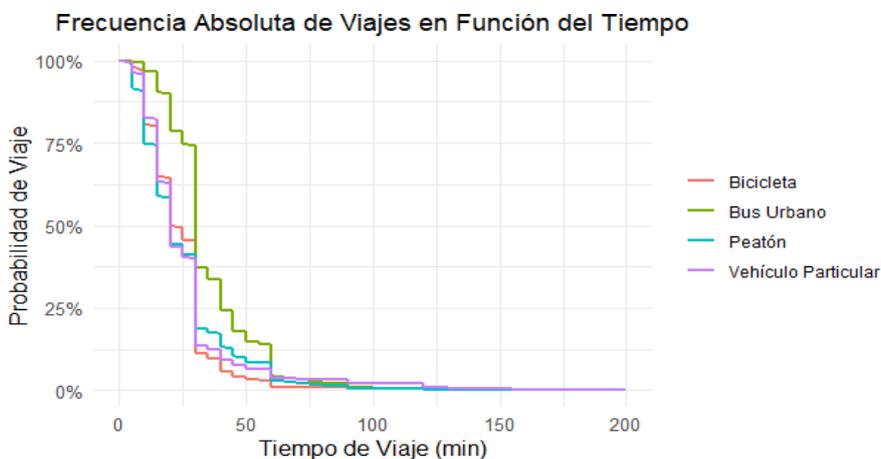


In original language Spanish

Figure 4. Absolute cumulative frequency of trips based on distance

Source: present research

Figure 4 shows the cumulative absolute frequency of trips according to distance traveled (km), categorized by mode of transport. It is evident that trips on foot (in red) dominate over the shortest distances (<5 km), accumulating the highest initial frequency due to the nature of this mode as a primary means of urban travel. Trips by bicycle (green) and urban bus (light blue) have lower accumulated frequencies over short distances, but remain relevant up to 20 km, which suggests an appropriate use for intermediate journeys. On the other hand, trips in private vehicles (purple) show a wider dispersion, extending up to 100 km, reflecting a preference for this mode on long-distance journeys.

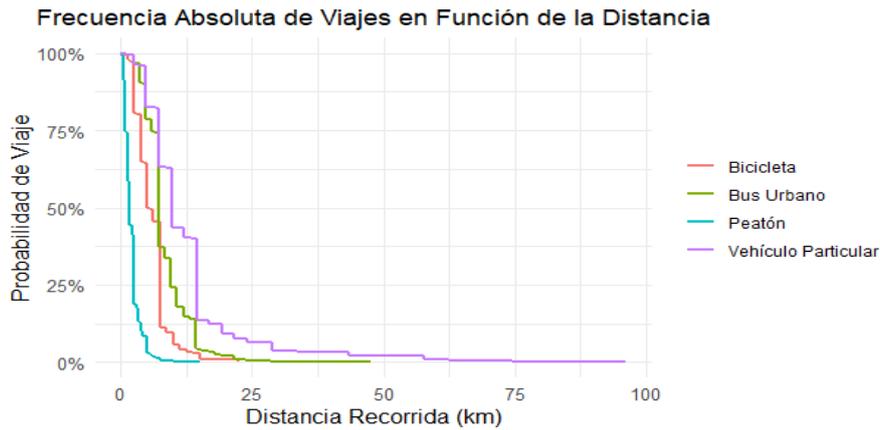


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Figure 5. Absolute frequency of trips as a function of time

Source: present research

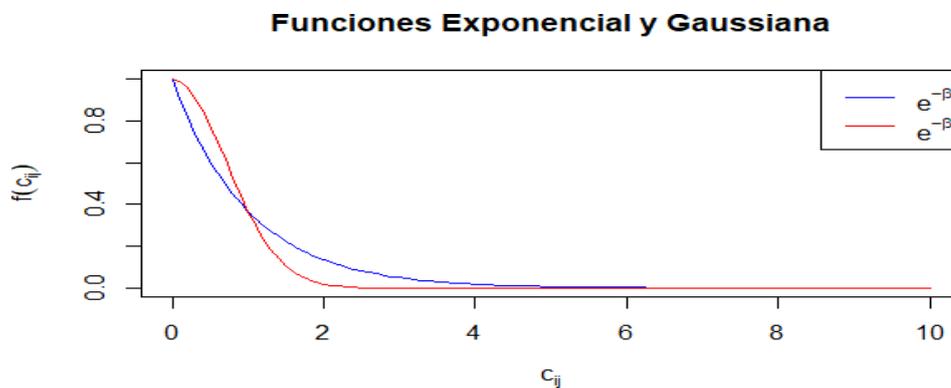
Figure 5 shows the relationship between the cumulative frequency of trips and the travel time (in minutes) according to the mode of transport. It is observed that transport on foot (sky blue) and bicycle (red) dominate on short journeys, accumulating about 75% of trips in less than 30 minutes, which reflects their functionality over limited urban distances. Urban buses (green) have a more uniform distribution, with a progressive increase in probability up to 60 minutes, which suggests frequent use for intermediate journeys. Private vehicles (purple), on the other hand, stand out for having a more dispersed travel time, reaching significant cumulative frequencies even after 60 minutes, which reflects their predominance on long journeys.



In original language Spanish  
 Figure 6. Absolute frequency of trips based on distance  
 Source: present research

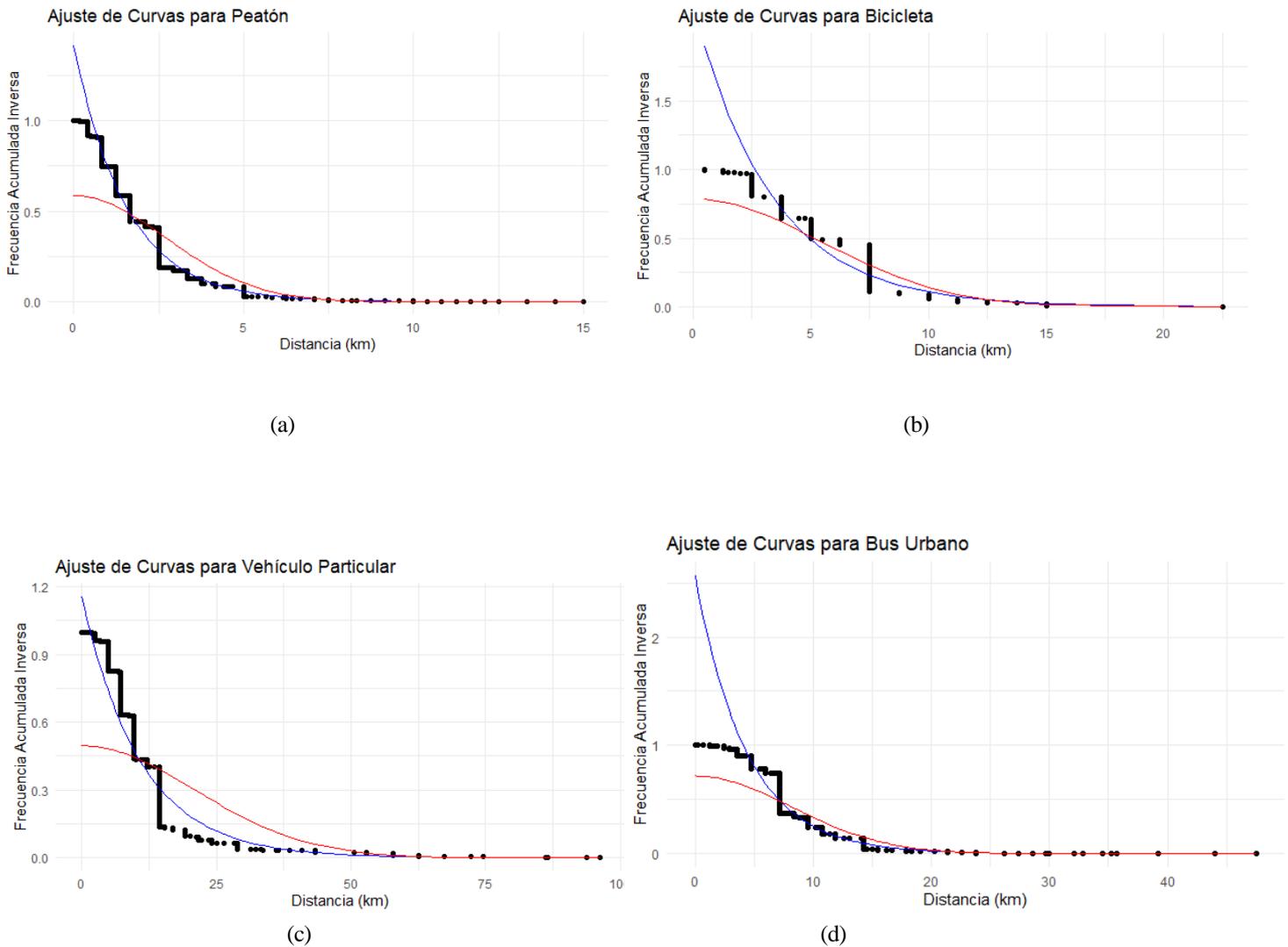
Figure 6 shows the graph of cumulative frequency of trips based on distance traveled (km), by type of transport. It is seen that walking (sky blue) predominates on very short journeys (< 5 km), and quickly reaches 100% accumulation, which confirms its exclusive use for short distances. The bicycle (red) and the urban bus (green) show an intermediate behavior between them, with significant accumulations on journeys up to 20 km, being effective especially on medium journeys. On the other hand, individual vehicles (purple) use to a greater extent up to distances of 100 km, where they show diffusion and little functionality on longer journeys. Mathematically, all distributions are reflected in an inverse exponential decay, characteristic of urban mobility phenomena where the probability decreases drastically with distance. This analysis highlights the importance of designing transport policies around the needs of each modal in relation to destinations and distances travelled, optimising at all times what is offered in terms of infrastructure and multimodal access.

When constructing and comparing the exponential and Gaussian functions in terms of  $c_{ij}$ , an independent variable associated with distance or cost. Both functions decrease rapidly, although they have significant differences. The blue line is the exponential function that exhibits a constant decay, which is appropriate for modeling contagion phenomena where the impact decreases proportionally to distance. On the other hand, the Gaussian function (red line) decreases more rapidly for low  $c_{ij}$  values, stabilizing near zero for high values, which makes it more restrictive in applications involving limited spatial interaction. This behavior suggests that the choice of function is related to the phenomenon to be modeled, with the Gaussian function being suitable for highly localized interactions, and the exponential suitable for more fuzzy relationships. See Figure 7.



In original language Spanish  
 Figure 7. Exponential and Gaussian functions  
 Source: present research

The set of graphs corresponding to the distance decay curves according to the mode of transport is presented below: pedestrian, bicycle, private vehicle and urban bus. The observed patterns typically behave as an inverse exponential distribution. Trips for pedestrians are more pronounced to decline, which means that most trips are short distances (<5 km). In the case of traveling by bicycle they extend a little more, but by 10 km they will have been significantly reduced as well, and the four modes of transport over shorter distances (private vehicles) show a greater dispersion over greater distances (>50 km), which indicates that it is applied on long journeys. On the other hand, the urban bus has an intermediate decay on journeys of up to 20 km, which reinforces its use on urban and suburban routes. Mathematically, the adjustments reveal the superiority of exponential and Gaussian functions in modeling these distributions. This analysis emphasizes the need to incorporate transport strategies that adapt to the types of predominant uses and the distance covered in each modality.

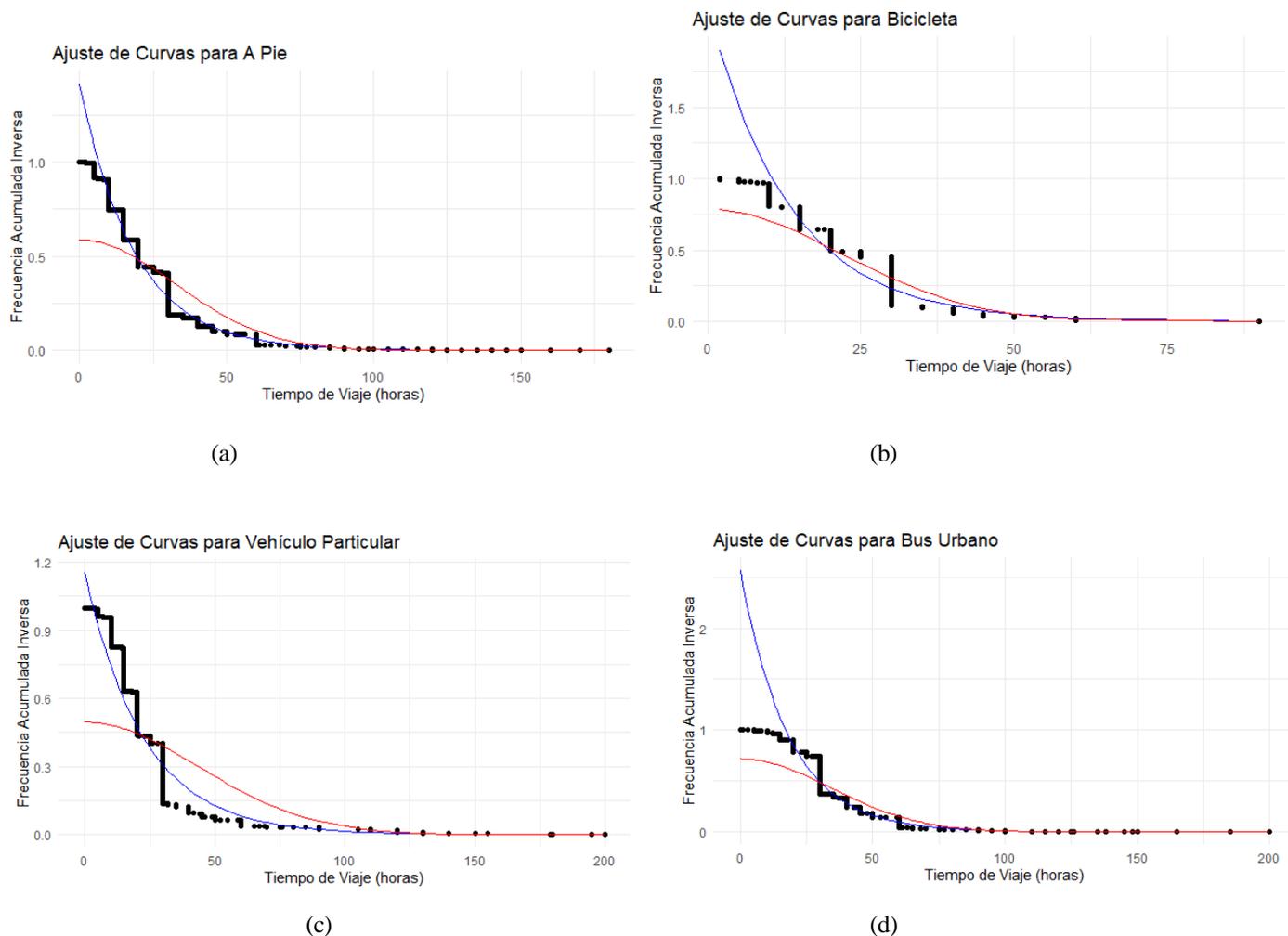


In original language Spanish

Figure 8. Distance decay curves for trips in different modes. (a) Pedestrian (b) Bicycle (c) Private vehicle (d) City bus  
 Source: present research

The graphs in Figure 9 represent the curves of travel time decay for four modes of transport: pedestrian, bicycle, private vehicle and urban bus. The analysis reveals that the pedestrian mode has a rapid decay, reaching low reverse accumulated frequencies in short times (<30 hours), which shows its use in limited distances in time and distance. On the other hand, bicycles and urban buses exhibit similar patterns, although with a wider range in intermediate times (up to 75 hours), highlighting their usefulness in medium-term journeys. In contrast, the private vehicle shows a more dispersed distribution,

with relevant frequencies even in high times (>100 hours), reflecting its predominance on long trips. Mathematically, these curves fit well with exponential and Gaussian models, the former being more suitable for local paths and the latter for more dispersed distributions. This analysis highlights the importance of designing optimal transport systems for each modality, according to the predominant duration of the journeys



In original language Spanish

Figure 9. Time decay curves for trips in different modes. (a) MIO (b) On foot (c) Bicycle and (d) Private vehicle.

Source: present research

Tables 1 and 2 present the statistical parameters for the adjustment of the exponential and Gaussian functions for the decay curves by distance (Table 1) and by travel time (Table 2). In both cases, it is observed that the exponential function achieves higher values of R2, with adjustments greater than 0.94 for most modes of transport, indicating a high level of accuracy in the representation of the data. The "pedestrian" and "bicycle" modes stand out for their lower errors (MSE and RMSE), which reflect more consistent patterns over short journeys. On the other hand, the Gaussian function shows a less robust performance, especially in the case of the private vehicle, where the error increases significantly due to the dispersion in distances and times. These differences highlight the usefulness of the exponential function for modeling urban mobility phenomena with high variability.

Distance decay functions  $\beta$  parameters for each mode

	Exponential Function				Gaussian function			
	Beta	R2	MSE	RMSE	Beta	R2	MSE	RMSE
Pedestrian	0.6427	0.9724	0.0042	0.0647	0.0692	0.7699	0.0328	0.1810
Bicycle	0.3009	0.8932	0.0217	0.1475	0.0169	0.8250	0.0141	0.1188
Private Vehicle	0.0916	0.8878	0.0111	0.1052	0.0011	0.6123	0.0577	0.2402
SETP Pasto	0.2320	0.9452	0.0229	0.1514	0.0076	0.7842	0.0250	0.1581

Table 1. Parameters  $\beta$  of the distance decay functions for each mode.

Source: present research

Time-decay function  $\beta$  parameters for each mode

	Exponential Function				Gaussian function			
	Beta	R2	MSE	RMSE	Beta	R2	MSE	RMSE
Pedestrian	0.0536	0.9724	0.0042	0.0647	0.0005	0.7699	0.0328	0.1810
Bicycle	0.0752	0.8932	0.0217	0.1475	0.0011	0.8250	0.0141	0.1188
Private Vehicle	0.0441	0.8878	0.0111	0.1052	0.0003	0.6123	0.0577	0.2402
SETP Pasto	0.0552	0.9452	0.0229	0.1514	0.0004	0.7842	0.0250	0.1581

Table 2. Parameters  $\beta$  of travel time decay functions for each mode.

Source: present research

Table 3 consolidates the  $\beta$  parameter of decay functions for time and distance, allowing key trends in trip impedance to be identified. It is observed that the  $\beta$  values are higher for the "pedestrian" and "bicycle" modes, reflecting a rapid decline in the probability of making long trips. In contrast, the urban bus and the private vehicle have lower values, evidencing greater resistance to decay on extended journeys.

	Impedance function	
	$\beta$ (Distance)	$\beta$ (Time)
Pedestrian	0.0536	0.00048
Bicycle	0.0752	0.00106
Private Vehicle	0.0441	0.00026
SETP Pasto	0.0552	0.00043

Table 3. Consolidation of the parameter  $\beta$  of the time and distance decay functions.

Source: present research

These results suggest that motorized modes are more suitable for long distances and long times, while active modes are preferred for short distances due to their efficiency and accessibility. Obtained for walking, a beta value of 0.0536 (distance) and 0.00048 (time) reflects a sharp decrease in accessibility as distance and time increases, highlighting the limitation of short journeys. The beta value of the bicycle: 0.0752 (with distance) and 0.00106 (with time) show, on the one hand, that it is more likely to travel greater distances than walking, but on the other that it is very sensitive to spatial friction. The beta values for private vehicles are low, 0.0441 (distance) and 0.00026 (time), indicating preference for longer journeys with more minimal friction. Finally, the beta values of public transport (0.0552 distance and 0.00043 time) combine the practicality of the service with accessibility. These beta values teach the critical role that distance and time friction play in travel behavior. Short distances in walking and cycling are restricted, below highlights the urgency of expanding the infrastructure to support these modes. While private vehicles and public transport offer greater connectivity, such connectivity must be balanced with policies that promote equity and sustainability. These decay functions and beta

parameter analysis can be integrated into urban planning and offer practical strategies for improving mobility and addressing spatial inequities in Pasto.

## 5. Conclusions and Recommendations

This study provides a solid foundation for understanding these mobility patterns in Pasto, emphasizing how distances, time, and modes of transportation are interrelated. Key development indicators based on estimated decay functions, intended to guide transport and land use planning.

The results indicate that active modes, such as walking and cycling, are limited to very high spatial friction, so active modes are inherently limited to short distances. This demonstrates the importance of integrating and strengthening this type of infrastructure so that it becomes a possibility for sustainable urban environments.

Private vehicles and public transport are more aimed at long journeys, however, their implementation must be balanced with policies that reduce their environmental impact and benefit sustainability. To make public transport competitive over intermediate distances, improving its efficiency is key.

To test the accuracy of the proposed models, they were validated with fit metrics, such as MSE and RMSE, and exponential functions are better for walking and for private vehicles, while Gaussian functions give a better representation of the preferences of bicycle and public transport users. The differences are necessary to adapt mobility strategies according to the modality.

For example, pedestrians travel, on average, up to 1 km in 12 minutes, bicycles cover 5 km in 20 minutes, private vehicles travel 15 km in 25 minutes, and public transport works efficiently over distances of 10 km and 40 minutes. These values are of great importance when optimizing infrastructure and transport services according to their characteristics for each mode.

The graphs of cumulative frequency of trips show the higher frequency of active modes on short journeys and of motorised modes on more varied routes. This reaffirms the need for integrated policies that strengthen infrastructure for sustainable modes (including public and private transport) to contribute to its optimization.

The  $\beta$  values obtained show how spatial friction affects travel decisions. Pedestrians have high  $\beta$  values, suggesting a rapid loss of accessibility with distance, while average values for cycling and collective transport offer optimal anchorage for longer distances. Extreme minutes through private vehicles due to their lower perceived friction in relation to other methods of transport.

This analysis demonstrates that integrating decay functions into urban planning can stimulate not only use to find mobility patterns but also to address spatial inequities with targeted strategies. This facilitates the closing of gaps in accessibility to basic services and in social and territorial cohesion.

The findings highlight the importance of comprehensive policies that complement infrastructure improvements with economic and regulatory stimulus, especially where the topography of cities can be extremely complex as in Pasto. The strategy for prioritization based on empirical data provides equity and sustainability in the transportation system.

In conclusion, the integration of functions of generation and analysis of  $\beta$  parameters in urban planning processes has configured over time a powerful tool to design mobility strategies that optimize connectivity, minimize spatial inequities and improve the quality of life of urban and suburban environments.

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