Invulnerability Analysis of Power Grid Considering Grounding Grid Faults Based on Gravity Method

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Abstract: How to use quantitative analysis methods to identify the importance of nodes in power grids is a hot topic in power grid research. Drawing on the theory of complex grid systems and considering the multi-attribute characteristics of network nodes, this paper proposes a method for evaluating the importance of network nodes and an improvement strategy for system invulnerability. Firstly, based on the gravity method, a node importance evaluation algorithm for the structural hole gravity model was constructed by comprehensively considering the node *H*-index, node position, and structural hole position, and the evaluation criteria for the evaluation algorithm were provided. At the same time, the impact of grounding grid faults on the importance analysis of system nodes was considered in the calculation and analysis. Secondly, based on the above analysis, optimization measures have been established to improve the invulnerability of the power grid. Finally, the rationality of the method proposed in this paper is verified through calculation and analysis under various operating conditions using IEEE 30 node examples and actual systems. Among them, the actual system utilizes the monitoring results of the grounding grid to calculate the importance of system nodes. The calculation results show that the algorithm proposed in this paper can better identify important nodes in the power system grid compared to the betweenness centrality method, and the system resilience enhancement strategy has good effectiveness and robustness.

Keywords: power grid, gravity model, node analysis, grounding grid, invulnerability

1. Introduction

Electricity is one of the most convenient and important sources of energy in modern society, The modern power system network is the basic prerequisite for the transmission, distribution and consumption of electrical energy. Coupled with the rapid development of smart grid construction, the power grid exhibits new characteristics, including more complex networks, a large number of nodes, and diverse forms of energy., and consequently, the power grid has suffered a number of serious incidents in recent years. For example, the Indian blackout that occurred in 2012 was beyond imagination in terms of its reach, duration, and destructiveness [1-2]. A large-scale blackout accident occurred in the Brazilian power grid in 2023 [3], which caused 25 states to be affected to varying degrees, with a total loss of load of 23368 MW. After the incident, it was found that through the analysis of the incident, these accidents were often caused by initial node glitches and small disturbances, and subsequently cascading effects leading to serious accidents. Therefore, it is particularly important to assess the critical nodes of the power system network and to improve the destruction resistance of the power system to cope with accidents.

The study of power grid node importance and system destruction resistance, the first need to study the system topology and the occurrence of faults in the transmission of the evolution of the mechanism, in this regard, the researcher mainly from these two aspects of the analysis: the abstract complex systems theory and the real reality of the systems. Based on the work of complex systems theory, researchers have analyzed the network distribution properties exhibited by the infrastructure [4-5]. Motter et al. introduced a cascading failure model based on the overload mechanism [6], which has become a valuable tool for studying the intrinsic mechanisms of system cascading failures and has gradually emerged as a research hotspot in recent years [7]. The cascading failure dynamics model based on the overload mechanism can help people explore and understand the internal structure of these complex systems, thereby enabling the control of complex systems in the direction desired by people. The classical cascading failure model does not consider changes in node load status and cannot reallocate loads in real-time based on node status. Currently, there are generally two types of evaluation methods for the importance of network system nodes: 1) analyzing the importance of nodes while maintaining the overall operational capability of the network; 2) analyzing the extent of network performance degradation after removing certain nodes, and measuring the importance of nodes through the reduction in network performance. Commonly used methods include node deletion method and contraction method [8]. Recently, researchers have proposed that the importance of nodes in a network is not only related to network topology [9-10] but also to the propagation mechanism. The above-mentioned evaluation methods for node importance only consider the static characteristics of the network, ignoring the impact of energy flow in the system network, and thus cannot well reflect the true importance of nodes. Based on the cascading failure dynamics model of the overload mechanism, this paper aims to propose a more targeted local load distribution model based on the evolution of node real-time status by leveraging the idea of node transmission power changes. By comprehensively considering multiple attribute characteristics of nodes, a corresponding node importance evaluation method is proposed.

Currently, both domestic and international research on invulnerability analysis and node importance evaluation of power system networks is still in its infancy. Most research has primarily concentrated on modeling and evaluating the importance of single factors influencing multiple power grids, with limited attention being paid to comprehensively assessing node importance from a multifaceted perspective that takes into account various factors. There are some research methods on how to evaluate the node importance of network systems, such as degree centrality, proximity centrality, betweenness centrality, and k-kernel decomposition methods. The centrality index considers the number of direct neighbors of a node, which is simple and intuitive, but treats each neighboring node as equally important, making it difficult to adapt to multiple scenarios. Proximity centrality and betweenness centrality both assume that information in the network propagates based on the shortest path, which is inconsistent with the actual multi scenario operation of network systems. The k-kernel decomposition method believes that the importance of network nodes is determined by their position in the network, therefore, it is impossible to distinguish the importance of nodes in the same shell. The *H*-index performs better than degree and kernel in many scenarios. Therefore, to overcome the above shortcomings, based on the gravity model, a comprehensive method integrating *S*-value, *H*-index, and structural hole features is proposed in the paper, considering factors such as inter node correlation, node position, and node transmission power, making the node importance evaluation results more comprehensive. Reference [11] identifies critical nodes in the power grid based on the power generation capacity of generator sets to determine the optimal restoration path after power outages. Reference [12] investigates the importance of grid nodes to optimize issues such as grid network reconfiguration. Reference [13] constructs an index system for identifying key nodes in power communication networks using the Analytic Hierarchy Process (AHP) combined with the topological structure of communication networks. By analyzing cascading failures in information-physical systems, References [14-15] evaluate the importance of nodes in information-energy coupled power networks, but the factors considered are relatively simple. Reference [16] presents an evaluation method for the importance of nodes in coupled distribution and communication networks, assessing the coupled network from an economic perspective.

The power system is a kind of complex network system, this paper will use the analysis theory of complex network system node importance to analyze the node importance of the power system network, the paper will take the number of node kernels as the index to measure the global importance of the node, integrate the H-index to redefine the node's importance, and combined with the structural hole characteristic of the node, design the node importance assessment algorithm of the gravitational model, and based on this, the invulnerability of the system is analyzed, and the invulnerability enhancement strategy is given. The rest of the organization of this manuscript is as follows: Section 2 elaborates on the importance and invulnerability analysis of power network system nodes, while Section 3 proposes a mathematical model for evaluating the importance of power network nodes and provides corresponding evaluation methods. Section 4 proposes enhancement strategies for improving the invulnerability of power system networks. Section 5 presents case verification analysis results, which validate the rationality and correctness of the method proposed in the article based on the results. Finally, concluding remarks are mentioned in Section 6.

2. The Importance and Invulnerability Analysis of Power Network System Nodes

The traditional invulnerability measurement of power system networks mostly considers the static characteristics of the network, focusing only on the impact of network topology or the changes in network functions after severe disturbances, while rarely taking into account the dynamic factors such as energy flow or information flow in the network that affect the invulnerability of the network. This paper proposes a node importance evaluation system for power network systems that takes into account both static and dynamic characteristics, as shown in Figure 1.

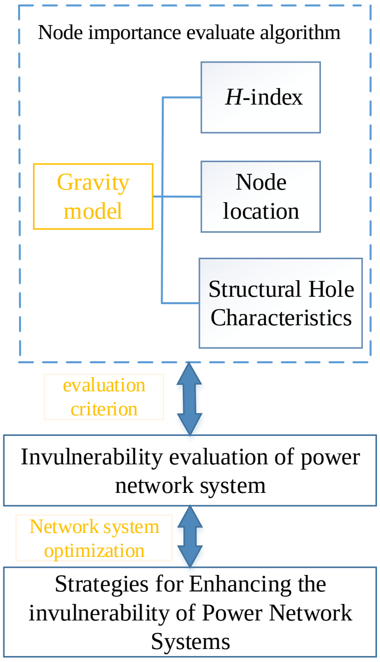


Figure. 1 node importance assessment system based on the gravity model

This evaluation system is divided into two layers: node importance evaluation and network system invulnerability enhancement strategies. The optimization strategy for enhancing the invulnerability of power networks is based on the node importance evaluation method and represents the centralized embodiment of the optimized operation of power network systems.

3. The Importance Evaluation of Power Network Systems Notes

3.1 Basic theoretical description

Complex power network systems can be represented in the form of graph theory $G=(N,A)$, where N represents the number of nodes in the network system, and A represents the set of edges between network nodes. In the study of describing the importance of network system nodes, different measurement methods have emerged, such as degree centrality, closeness centrality, H-index, and betweenness centrality [17-19]. The degree centrality method is relatively simple and intuitive, representing the number of adjacencies of a network node. The degree of node i can be expressed as:

$$\alpha_i = \sum_{j=1}^{N} a_{ij} \tag{1}$$

where a_{ij} represents the line connection relationship between node i and node j. If there is a line connection between node i and node j, then a_{ij}=1; otherwise, a_{ij}=0. N is the number of nodes in the network system.

The degree centrality index reflects the direct relationship between nodes. In the power system, the more lines connected to a node, the larger the node degree α_i value, and the more important the node is. However, this index only considers the local information of the node, which is too one-sided and cannot fully reflect other information of the network system nodes (such as generator nodes). It is a local centrality index.

Based on the average distance between a node and other nodes, closeness centrality believes that the greater the average distance, the higher the importance of the node. It provides an expression to measure the importance of node i:

$$\beta_i = \frac{\sum_{j=1}^{M} d_{ij}}{M} \tag{2}$$

where d_{ij} represents the distance between node i and node j, and M represents the number of nodes in the network system. To consider the actual operation of the power system, in the calculation of the distance between nodes in the power system network, d_{ij} is the electrical distance between node i and node j. Some researchers use the relationship between power changes and changes in node voltage to establish the electrical distance between nodes [20]. In this section, based on the Euclidean distance of node voltage sensitivity, the electrical distance between different nodes is constructed, and the calculation expression is:

$$d_{ij} = \frac{1}{2}\sum_{k=1}^{M-1}\left(\sigma_{ik} - \sigma_{kj}\right)^2 \tag{3}$$

$$\sigma_{ik} = -\lg\left|\frac{S_{ik}}{\max\limits_{k} S_{ik}}\right| \tag{4a}$$

$$\sigma_{kj} = -\lg\left|\frac{S_{kj}}{\max\limits_{j} S_{kj}}\right| \tag{4b}$$

where d_{ij} represents the electrical distance between node i and node j; S_{ik} is the element in the kth column and ith row of the sensitivity matrix; $\max\limits_{k} S_{ik}$ is the maximum value among the elements in the kth column of the sensitivity matrix; M is the number of nodes in the network system.

The H-index was originally used to represent the value of the highest number of citations for a scholar's papers [21], and the size of this value reflects the scholar's academic achievements. Later, it was applied to complex network systems, where the H-index of a node is defined as the degrees of the node's h adjacent nodes are all no less than h, reflecting the importance of the node in the network system. In a network system, if a node has an H-index of h, it can be expressed as:

$$H_i = \psi\left(k_{i1}, k_{i2}, \ldots, k_{is}\right) \tag{5}$$

where ψ represents an operator, H_i represents the H-index value of node i, k_{is} represents the degree of the sth adjacent node of node i, and s represents the number of adjacent nodes of node i. When evaluating the importance of nodes, the comprehensive performance of the H-index is superior to the degree [22].

The structural hole approach refers to the gaps in the network system. If there is no direct connection between two nodes, they must rely on a third party to form a connection. Then, the third party occupies a structural hole in the network system. The fewer the existing third parties, the more important the third party that can serve as a connection, and the higher the importance of that node. Applying the structural hole theory method to measure the importance of nodes can more effectively reflect alternative routes and produce a more reasonable evaluation. Let k be the common adjacent node of node i and node j in the power network system. Node i can be directly connected to node j, or it can be connected to node j through node k. Then, the quantitative expression for structural hole constraint is as follows:

$$c_i = \sum_{j}\left(w_{ij} + \sum_{k(\neq i,j)} w_{ik} w_{kj}\right)^2 \tag{6}$$

where w_{ij} represents the proportion of the connection between node i and node j to all the external connections of node i, and node k indicates that it is adjacent to both node i and node j. The calculation formula for w_{ij} is as follows:

$$w_{ij} = z_{ij} \Big/ \sum_{j\in\Gamma(i)} z_{ij} \tag{7}$$

where $\Gamma(i)$ represents the set of nodes adjacent to node i. When node i and node j are adjacent, z_{ij}=1; otherwise, z_{ij}=0.

In the power network system, the more structural holes a node occupies, the less it is constrained by other nodes, and the smaller the structural hole constraint is. Below, we illustrate the evaluation of node importance using the structural hole theory method through a simple network node model, as shown in Figure 2. In Figure 2, node 1 is adjacent to three other nodes, so w_{12}=w_{13}=1/3. Node 3 is adjacent to both node 1 and node 2, so w_{32}=1/2. Then, the quantitative value of the structural hole constraint for node 1 in the node network model is c_1=0.611.

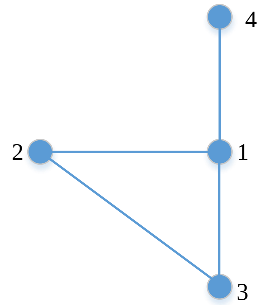


Figure. 2 Diagram of Simple Network Node Relationships

3.2 Analysis of the Importance of Power Network Nodes Based on Gravity Model

The H-index reflects the importance of nodes from the clustering level, while the structural hole theory reflects the importance from the intermediary level. In order to draw on the advantages of different methods and make the evaluation results more accurate, based on the gravity model [23], this paper uses the H-index and structural hole theory to weigh the nodes of the power system network, considers the multiple attribute characteristics of nodes, and obtains a comprehensive node importance evaluation method. Among them, the S value of the power network system represents the hierarchical result of nodes after decomposition, which reflects the importance of nodes from the perspective of network degree. The H index reflects the importance from the clustering level, and the structural hole theory reflects the importance from the intermediary level. As shown in Figure 2, the S value of nodes 1, 2, and 3 is 2, and the S value of node 4 is 1.

The position of a node in the power network is an important attribute of the node. The location information not only includes the centrality of global information but also the structural hole position of local information. When a node has a high H-index and occupies many structural holes at the same time, it often has a greater influence on the entire network system. Based on the above analysis, this paper constructs an evaluation and analysis method for the importance of nodes in the power system network. Its basic idea is to comprehensively consider the local topological information and global location information of nodes in the network system, and integrate the structural hole characteristics of nodes, so as to reduce the negative impact of falsely high importance of pseudo-core nodes on the correctness of algorithm ranking. This idea can effectively use the interaction force between nodes and neighboring nodes to describe the importance of nodes, and can comprehensively reflect the importance of nodes in the power system network from different angles.

For any node i and node j in the system network, utilize node S value and H index, the connection $B(i, j)$ generated between them is expressed by the following:

$$B(i,j) = \frac{(S_i + \lambda h_i)(S_j + \lambda h_j)}{d_{ij}^2} \tag{8}$$

where S_i and S_j are the S values of node i and node j respectively, λ is the equilibrium factor, $\lambda = \bar{d}\bar{S}/\bar{h}$, $\bar{d}$ is the average degree of all nodes, $\bar{S}$ is the average S value of all nodes, and $\bar{h}$ is the average H-index value of all nodes, h_i and h_j are the H-index values of node i and node j respectively, and d_{ij} is described in (2).

Based on this, the structural hole constraint is introduced to reduce the result error caused by the inflated importance of pseudo-core nodes, and combined with the influence of system network nodes in the node domain, the final evaluation result of node importance is obtained as shown in the following:

$$D(i) = \sum_{d_{ij} \in \phi(i)} e^{-c_i} \frac{(S_i + \lambda h_i)(S_j + \lambda h_j)}{d_{ij}^2} \tag{9}$$

where $\Phi(i)$ is the set of nodes whose distance to node i is less than or equal to a given value. In order to reduce the complexity of the algorithm, this given value is set to 3 in this paper[24]; other parameters are described in (6) and (8).

In the simple network node relationship shown in Figure 1, based on the calculation using (8), the influence value between node 1 and node 2 is $B(1, 2)$=16, the influence value between node 1 and node 3 is $B(1, 3)$=16, and the influence value between node 1 and node 4 is $B(1, 4)$=8. The quantitative value of the structural hole constraint of node 1 is 0.611, and the evaluation result of the importance of node 1 in this network system is: $D(1)$=10.86.

In the operation of power systems, generally, the power transmitted by different nodes and the lines between them varies, making the importance of different nodes to the overall system operation different. When calculating the adjacency relationship between nodes, only considering whether the nodes are adjacent to each other without taking into account the influence of the power transmitted through the lines between nodes is one-sided in reflecting the importance of nodes. Furthermore, in order to comprehensively reflect the influence of the adjacency relationship between different nodes on the importance of nodes, this paper presents the following algorithm for calculating the adjacency relationship between node i and node j:

$$z'_{ij} = \begin{cases} \theta_1 + \theta_2 \dfrac{S_{ij}}{S_\Sigma}, \ as \text{node i and node j are adjacent to each other} \\ 0, \text{others} \end{cases} \tag{10}$$

where θ_1 and θ_2 are parameters, and in this paper's calculations, θ_1=θ_2=0.5 are taken. S_{ij} represents the transmission power between node i and node j, and S_Σ represents the total power of the network system.

Therefore, the calculation formula for the adjacency relationship between two nodes, derived from (7), is improved as follows:

$$w_{ij} = z'_{ij} \Big/ \sum_{j \in \Gamma(i)} z'_{ij} \tag{11}$$

where z'_{ij} is described in (10).

Based on the above analysis, in calculating the adjacency relationship between different nodes using (7), this paper proposes to improve it with the algorithms in (10) and (11). The improved algorithm given in (11) in this paper can not only reflect the topological characteristics of the power system network but also reflect the system operation mode, enabling a more comprehensive and accurate reflection of the importance of different nodes.

3.3 Evaluation Standard Analysis

This paper uses the invulnerability of the network system to evaluate the accuracy of the node importance evaluation method proposed in the paper. The main idea of this part is: when a node in the power system network fails, the network system can still maintain its operational capability, that is, the more important a node is, the greater its impact on the invulnerability of the network system. If the node is removed, the decline in the invulnerability of the network system will be more obvious. After the power network system is disturbed, the higher its connectivity, the lower the probability of system collapse. Therefore, network connectivity is an important indicator to measure the degree of invulnerability of the power network system. Therefore, this part uses the relative size of network efficiency [25] and the largest connected subgraph [26] to evaluate the effectiveness and accuracy of the node importance evaluation method proposed in the paper.

On a global scale, network efficiency quantifies the energy transmission capacity of power system networks; on a local scale, it quantifies the network's ability to resist faults and disturbances. The calculation of network efficiency E_1 is shown in the following:

$$E_1 = \frac{1}{N\left(N-1\right)} \sum_{i \neq j \in G} \frac{1}{t_{ij}} \tag{12}$$

where N represents the number of nodes in the power system network, G represents the set of nodes in the power system network, and t_{ij} represents the shortest path between node i and node j.

The largest connected subgraph, also known as a connected component, is an important indicator for measuring the reliability of power system networks. The larger its value, the stronger the invulnerability of the power network. This paper adopts the following definition:

$$E_2 = \frac{C_1}{C_2} \tag{13}$$

where C_1 represents the number of nodes contained in the largest connected subgraph, and C_2 represents the total number of nodes in the network system.

Based on the above analysis, this section presents the formula for measuring the invulnerability of the power network system as follows:

$$R = E_1 * E_2 \tag{14}$$

According to the evaluation algorithm of node importance in the previous text, the nodes are sorted from high to low in terms of their importance, and the nodes (or edges) are removed in this order. The invulnerability of the network system is calculated for each removal, and (14) can be applied to evaluate the node importance evaluation method proposed in this paper.

4. Strategies for Improving the Invulnerability of Power Network Systems

4.1 Objective Function

The key to optimizing the invulnerability of network system operations lies in the design of the topological structure and the selection of appropriate operation modes, in order to maximize the invulnerability of the network system. In general, the invulnerability of a network system needs to consider various factors such as nodes and operation modes within the network. This section presents the idea of enhancing the invulnerability of power system networks as follows: by adjusting the operation modes of power system networks (such as regulating generator output or the power of flexible loads at nodes), the relationships between line nodes can be adjusted, thereby achieving the goal of improving the invulnerability of the power system. In other words, the value of (14) is increased through optimization strategies. Therefore, when studying optimization and enhancement strategies for network system invulnerability, this paper comprehensively considers factors such as network system nodes and operation modes, i.e., the network system is able to maximize its ability to withstand disruptive events. This section presents the objective function expression for invulnerability optimization and enhancement as follows:

$$\max R = E_1 * E_2 \tag{15}$$

where E_1 and E_2 are described in (12) and (13) respectively.

4.2 Constraints of the Power System Grid

（1）System grid power flow constraints

To ensure the secure and stable operation of the power system grid, the optimization and adjustment process must guarantee that the active power output of all generator nodes in the system is balanced with the total load demand of the system, it needs to satisfy the power flow constraint equation. The constraint conditions of the equation can be expressed as:

$$P_i^G = P_i^{load} + U_i \sum_{j=1}^{N} U_j \left(G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij} \right) \tag{16}$$

$$Q_i^G = Q_i^{load} + U_i \sum_{j=1}^{N} U_j \left(G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij} \right) \tag{17}$$

where P_i^G and Q_i^G represent the active power and reactive power of generator node *i*, respectively; P_i^{load} and Q_i^{load} represent the active power and reactive power of load node *i*, respectively; U_i and U_j are the voltage magnitudes of node *i* and node *j*, respectively; θ_{ij} is the phase angle difference between system node *i* and node *j*; and G_{ij} and B_{ij} are the conductance and susceptance between node *i* and node *j*, respectively.

（2）Node power constraint

Output limits constraint of generator nodes. The output of generator node *i* cannot be less than its minimum power generation capacity, nor can it exceed its maximum power generation capacity. It needs to satisfy the following constraints:

$$P_i^{G,\min} \le P_i^G \le P_i^{G,\max} \tag{18}$$

where $P_i^{G,\min}$ and $P_i^{G,\max}$ represent the lower and upper limits of the output of generator node *i*, respectively.

Transmission power constraint of lines. Each line has a limited power carrying capacity, and if the transmission power on the line exceeds the limit, "congestion" will occur. Therefore, the transmission power P_{ij} of branch *i-j* needs to satisfy the following constraint conditions:

$$P_{ij} \le P_{ij}^{\max} \tag{19}$$

where $P_{ij}^{\max}$ represents the power limit that can flow through the branch *i-j*.

For flexible load node *j*, the load power of node *j* can be composed of rigid load and flexible adjustable load, and the expression is:

$$P_j^{load} = P_j^c + P_j^f \tag{20}$$

where P_j^{load} represents the load power of node *j*, P_j^c represents the rigid load power of node *j*, and P_j^f represents the flexible adjustable load power of node *j*.

5. Case Verification Analysis

5.1 Analysis of IEEE 30 Node System Calculation Example

5.1.1 Case description

This section verifies the proposed method in the article using the IEEE 30-bus system as the test system. Its topological structure is shown in Figure 3, and Table 1 provides some parameters of the test system. The simulation test is based on the MATLAB platform, and various operating conditions are considered in the calculation process. Multiple methods are applied for comparative analysis to verify the effectiveness and correctness of the analysis method proposed in this paper.

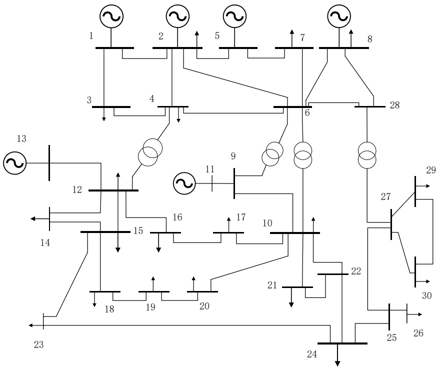


Figure. 3 Structure diagram of the IEEE-30 bus system

Table1 Partial line parameters of the system

Number	Initial Node	Terminal Node	Branch Resistance/p.u.	Branch Reactance/p.u.	1/2 Charging Susceptance/p.u.
1	1	2	0.0192	0.0575	0.0264
2	1	3	0.0452	0.1852	0.0204
3	2	4	0.0570	0.1737	0.0184
4	3	4	0.0132	0.0379	0.0042
5	2	5	0.0472	0.1983	0.0209
6	2	6	0.0581	0.1763	0.0187
7	4	6	0.0119	0.0414	0.0045
8	5	7	0.0460	0.1160	0.0102
9	6	7	0.0267	0.0820	0.0085
10	6	8	0.0120	0.0420	0.0045

5.1.2 Calculation of component importance

Scenario 1: Assuming a sustained high ambient temperature, with a sudden increase in load at certain nodes (specifically, a 20% increase in power load at Nodes 14 and 29, while the power consumption remains unchanged at other load nodes), it is necessary to calculate the importance of each node to provide technical reference for power dispatching operations.

Based on the theoretical method proposed above, the importance values of nodes can be calculated. For comparative analysis, this section adopts two scenarios: before and after the load increase, to conduct separate calculations and analyses, and to compare the changes in node importance under different scenarios. Before the load increase, the calculation results of the importance of some nodes are shown in Table 2. After the load power increases, the calculation results of the importance of some nodes are shown in Table 3.

Table 2 Importance Values of Selected Nodes in the Power System (Before Load Increase)

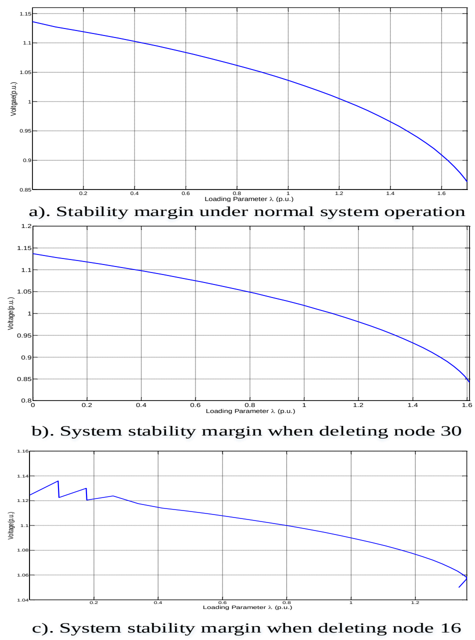
Node	Importance Value
4	12.72
13	11.58
12	10.79
14	9.88
16	10.32

27	10.64
28	10.93
29	9.81
30	9.55

Table 3 Importance Values of Selected Nodes in the Power System (After Load Power Increase)

Node	Importance Value
4	12.31
13	11.97
12	11.12
14	10.67
16	10.82
27	11.23
28	11.41
29	10.73
30	9.41

The calculation results from Tables 2 and 3 indicate that the importance values of Nodes 12 and 14 increased after the load power increase, primarily due to the increased transmission power. The increase in importance of Nodes 27 and 29 is mainly attributed to the increased load power at Node 29, which in turn led to an increase in the transmission power between Nodes 27 and 29. Nodes 4, 13, and 30 exhibited minimal changes in importance, as the increased load power at Nodes 14 and 29 had little impact on these nodal regions. The importance values calculated in Tables 2 and 3 accurately reflect the operational status of the power system network, showcasing the significance of different nodes within the overall system network under various operating conditions. To further validate the proposed node importance assessment algorithm, the following analysis measures the importance of different nodes through the system stability margin. Based on the calculation results from Table 2, Node 16 has a higher importance than Node 30. Now, Nodes 16 and 30 are separately removed from the system, and the system stability margins are calculated for each case. The results are presented in Figure 4.



a). Stability margin under normal system operation

b). System stability margin when deleting node 30

c). System stability margin when deleting node 16

Figure. 4 System Stability Margin under Different Operating Conditions

In Figure 4, Figure a represents the stability margin of the system under normal operation, Figure b shows the stability margin when Node 30 is removed from the system, and Figure c depicts the stability margin when Node 16 is removed from the system. The stability margin in Figure c is significantly lower than that in Figure b, indicating that the importance of Node 16 within the entire system is higher than that of Node 30. This further validates the correctness of the node importance assessment algorithm proposed in this paper.

Scenario 2: A severe typhoon occurs in the external environment, resulting in the disruption of Lines 16-17. The power system dispatch department needs to adjust the operation mode and take preventive measures in advance for certain critical nodes, requiring the calculation of the importance of each node.

Using the theoretical method proposed earlier, the importance values of nodes can be calculated. For comparative analysis, this section adopts two operating conditions: before and after the disruption of Lines 16-17, respectively performing calculations and analyses to compare the changes in node importance under different operating conditions. Prior to the line disruption, the calculation results for the importance of some nodes are shown in Table 4. After the line disruption, the calculation results for the importance of some nodes are presented in Table 5.

Table 4 Importance Values of Partial Nodes in the Power Network Before Line Disruption

Node	Importance Value
10	11.84
13	11.52
12	11.13
14	9.47
16	10.61
17	9.89
18	10.02
20	10.12
21	9.76

Table 5 Importance Values of Partial Nodes in the Network After Line Disruption

Node	Importance Value
10	12.34
13	11.74
12	11.68
14	9.56
16	10.03
17	9.74
18	10.01
20	10.16
21	9.78

The calculation results from Tables 4 and 5 indicate that the importance of different nodes has changed before and after the disruption of Lines 16-17. Prior to the disruption, Node 16 transmitted power to Node 17, making Node 16 more important than Node 17. After the disruption, the importance value of Node 10 increased significantly, primarily because the load power of Node 17, which was previously supplied by interconnection Line 16-17, shifted to be provided by Lines 10-17. This increase in transmission power of Node 10 led to an elevation in its importance. In contrast, the importance values of Nodes 18, 20, and 21 changed very little, as the disruption of Lines 16-17 had minimal impact on the transmission power of these three nodes. The results of the importance calculations in Tables 4 and 5 effectively reflect the actual operating conditions of the power system network and verify the rationality and correctness of the node importance assessment algorithm proposed in this paper.

Based on the node importance evaluation method proposed in the text (Equation (14)), different nodes (as mentioned in Table 4) were removed from the network system, and the invulnerability performance of the network system was calculated. The results are presented in Table 6.

Table 6 Invulnerability Calculation Results of the Network System

Removing nodes	Grid system invulnerability
10	0.825
13	0.830
12	0.852
14	0.927
16	0.894
17	0.921
18	0.916
20	0.911
21	0.922

Table 6 calculates the invulnerability index values of the network system after removing different nodes, which reflect the invulnerability performance of the system. When Node 10 is removed, the invulnerability of the system decreases most significantly, indicating that this node has a higher importance. In contrast, when Node 14 is removed, the invulnerability of the system decreases less, suggesting that this node has a lower importance. This finding is consistent with the calculation results in Table 4 and reasonably validates the correctness of the node importance evaluation algorithm proposed in this paper.

The calculated importance index of power system network nodes can not only reflect the operating status of the power system or equipment but also intuitively show the risk-bearing capabilities of different nodes or lines, providing a foundation for power system dispatch and control. Therefore, compared with the existing importance evaluation methods and indicators [14-16], the method proposed in this paper avoids solving the model of failure probability, reducing the computational complexity. Meanwhile, compared with the existing single or static indicators, the proposed importance evaluation method possesses richer and more comprehensive information, which can provide a certain degree of reference for power dispatch departments.

In summary, the node importance evaluation algorithm proposed in this paper embodies the varying degrees of importance of different nodes in the power network system, and can also visually reflect the changes in the overall invulnerability of the network system as nodes are removed.

5.1.3 The optimized calculation results of the invulnerability enhancement strategy

To verify the advantages of the invulnerability enhancement strategy proposed in this paper, under the same operating conditions, a comparative analysis is conducted between the invulnerability of the system before and after optimization using our proposed strategy. Firstly, the importance of nodes in the IEEE 30-node power system is calculated, and the importance values of 5 nodes are provided. Based on this, each node is removed separately, and then the invulnerability of the network system is calculated. The calculation results are shown in Table 7.

Table 7 Calculation Results of Node Importance and System Invulnerability

Node	Importance Value	Grid system invulnerability
6	12. 43	0.805
9	12. 87	0.802

20	10.16	0.903
23	9.54	0.924
25	10.02	0.918

The calculation results in Table 7 indicate that Node 9 is a critical node with a relatively high importance value. If Node 9 is removed, it will significantly impact the system, causing a substantial decrease in the network invulnerability to 0.875. In contrast, Node 23 is a non-critical node with a lower importance value of 9.54. If Node 23 is removed, the network invulnerability will slightly decrease to 0.924, having a minimal impact on the overall network system. In practical power systems, Node 9 is a generator node that significantly influences the system, while Node 23 is a load node with minimal impact on the system. The calculation results in Table 7 are consistent with the operational conditions of the power network system, verifying the correctness and rationality of the calculation method proposed in this paper.

To further validate the effectiveness of the optimization strategy proposed in this paper for enhancing the invulnerability of power network system operations, we now consider the invulnerability and performance of the optimization strategy under attack scenarios. When a power network system is subjected to attacks, it can cause some nodes to malfunction. In this section, we simulate attacks on the network system and optimize it according to the invulnerability optimization strategy proposed in this paper. We then conduct a comparative analysis of the invulnerability of the network system before and after implementing the optimization strategy under the same operating conditions. The calculation results are shown in Figure 5.

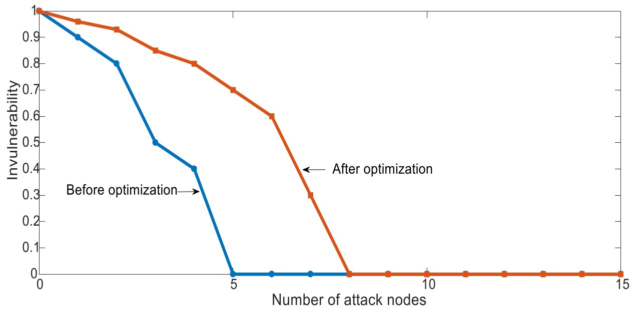


Figure. 5 Invulnerability of the Network System under Attack Conditions

As can be seen from Figure 5, the invulnerability of the network system declines when subjected to attacks. Before implementing the optimization strategy, as the number of attacked nodes increases, the majority of the remaining nodes in the network become isolated when the number of attacked nodes reaches 6, leading to a sharp decline in the system's invulnerability (as shown by the red line in Figure 5). After optimizing the network system by adjusting the tie-line structure of the attacked nodes and the power transmission of related lines, the invulnerability of the network system still remains at a value of 0.6 when the number of attacked nodes reaches 6, with fewer remaining nodes becoming isolated, and the impact on the network system remains controllable. However, as the number of attacked nodes continues to increase, the maximum transmission capacity limitations of the surrounding neighboring nodes and lines are unable to accommodate the high loads from the attacked nodes and lines, triggering cascading failures that further reduce the invulnerability of the network system until it reaches zero. Comparing the blue and red lines in Figure 5, it is found that attacking only one node in the network system does not significantly impact the connectivity and invulnerability of the network. However, as the cumulative number of attacked nodes increases, the optimization and control strategy proposed in this paper effectively enhances the network system's ability to withstand attacks, particularly in cases of severe attacks, improving the invulnerability of the network system.

5.2 Analysis of Actual Grid Examples

5.2.1 Example description

This section takes the actual power system in a certain region of East China Power Grid as an example for calculation and analysis. The system contains 63 nodes, 88 edges, and different types of loads such as industrial load, commercial load, and agricultural load. At the same time, the system contains renewable energy such as wind power and photovoltaic power generation. The grid system structure diagram is shown in Figure 6.

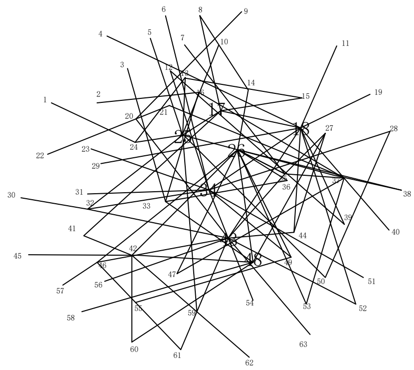


Figure. 6 Network system architecture diagram

5.2.2 Component importance analysis

To verify the effectiveness of the proposed method, this section uses the betweenness centrality method for comparative calculation and analysis [27-29]. The actual working condition is that the grounding grid of system node 32 has malfunctioned, and the grounding grid of node 32 is shown in Figure 7. The grounding grid generates a large grounding circulation (current density as shown in Figure 8), and the power dispatch department requires maintenance to be arranged. Currently, system node 32 is out of operation.

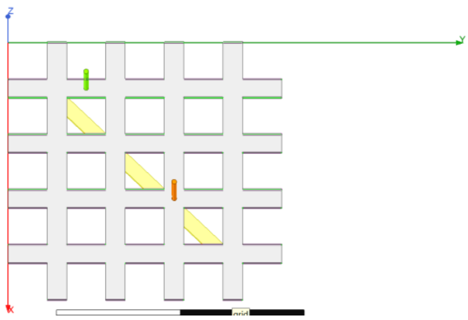


Figure. 7 Grounding Grid Model for Node 32

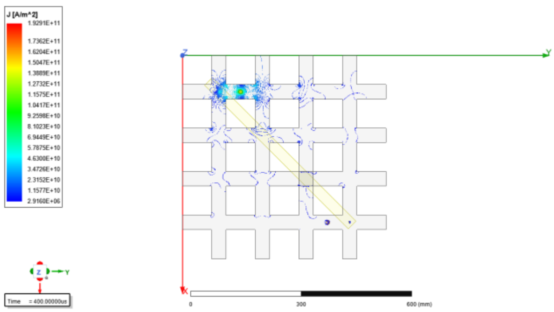


Figure. 8 Distribution diagram of grounding grid current density

Calculate and analyze the impact of grounding grid faults on the importance of system nodes, and analyze the changes in node importance before and after the occurrence of faults. No faults occurred, and the calculation results of the importance of some nodes are shown in Table 8. After a grounding grid fault occurred, the importance calculation results of some nodes are shown in Table 9.

Table 8 Importance values of some nodes in the power system (no faults have occurred)

Node	The proposed method in this article	Betweenness
17	12.03	11.50
18	12.89	12.15
23	9.15	8.66
27	10.37	9.45
30	9.22	8.73
32	10.93	11.47
34	12.96	12.11
43	13.48	12.84
60	11.73	10.28
61	10.98	10.29

The calculation results in Table 8 indicate that different methods have different results in calculating node importance. The method proposed in this article has importance levels of 11.73 and 10.98 for nodes 60 and 61, respectively. This is because the connection line structure of the two nodes is basically the same, but the transmitted power differs greatly. Node 60 transmits more power, while node 61 transmits less. From the perspective of system grid operation, node 60 is more important. However, in the betweenness centrality method, the importance of nodes 60 and 61 is basically equal, which does not fully match the actual operational requirements. This is because the betweenness centrality method focuses on the topological information of nodes in the entire grid system to evaluate their importance. Therefore, the calculation results of the betweenness centrality method are relatively one-sided and cannot fully and accurately reflect the importance in the entire grid system.

Table 9 Node importance value(after a fault occurs)

Node	The proposed method in this article	Betweenness
17	12.04	11.50
18	12.89	12.15
23	9.15	8.66
27	10.34	9.45
30	9.22	8.75
34	12.45	12.11
43	13.48	12.84
60	11.73	10.28
61	10.98	10.29

The calculation results from Tables 8 and 9 indicate that before and after node 32 exits operation, the importance of different nodes has changed, and the calculation of their importance should also change. However, the calculation results of the betweenness centrality method have not changed much, because the positions and system structures of other nodes have not changed much, but the importance of the nodes in the actual operating system has changed to some extent. Comparative calculations show that the method proposed in this article can more comprehensively and accurately reflect the actual operation of the system. The betweenness centrality method cannot fully adapt to the changes in network systems, and the calculation results cannot fully reflect the real situation.

Remove different nodes from the network system (as described in Table 8) and calculate the resilience performance of the network system. The calculation results are shown in Table 10.

Table 10 Calculation results of grid system Invulnerability

Removing nodes	Grid invulnerability
17	0.883
18	0.872
23	0.964
27	0.934
30	0.955
32	0.921
34	0.863
43	0.817
60	0.902
61	0.915

Table 10 calculates the resilience performance indicators of the grid system after removing different nodes. These values reflect the resilience performance of the system. If node 43 is removed, the resilience of the system decreases the most significantly, indicating that the importance of the node is higher; If node 23 is removed, the system's resilience decreases less, indicating that the node's importance is lower. The calculation results in Table 8 are consistent, which reasonably verifies the correctness of the node importance evaluation algorithm proposed in the article.

5.3 Evaluation and analysis of node importance in power grid

The numerical calculation analysis indicates that the IEEE 30-node system network exhibits strong invulnerability when facing relatively mild attacks but shows a certain degree of vulnerability when the attack intensity increases. In cases of cascading failures of some nodes, this vulnerability becomes more pronounced. As attacked nodes fail, the network's invulnerability declines sharply, and when the number of failed nodes reaches a certain amount, the network can be split into multiple isolated subnets, resulting in complete grid paralysis.

The actual system calculation results in a certain area of East China Power Grid show that the node importance of the grid system is not only related to the node position, but also to factors such as node transmission power, electrical distance between nodes, and structural hole characteristics. Accurately evaluating the importance of nodes requires a comprehensive consideration of the influence of multiple factors.

Therefore, when formulating emergency response strategies for power grids, it is essential to avoid the occurrence of cascading failures. Adequate protection measures for power grid nodes, especially critical nodes, should be implemented. On the basis of considering economic benefits, appropriately increasing the transmission capacity of nodes and lines can enhance the coupling relationship between critical nodes and other nodes, improving their ability to transfer loads to other nodes in case of overload. Additionally, after the occurrence of cascading failure, strict control should be exercised to prevent the load from shifting to nodes with lower importance values, in order to avoid situations where nodes bearing the load lack sufficient alternative paths to continue load distribution, which could lead to even larger-scale failures.

6. Conclusions

Power grids are complex with numerous nodes, posing significant challenges to grid dispatching and operation. Accurately identifying the importance of different nodes in the system grid is of great significance for improving the safe operation of the power grid. Based on the gravity model, this paper designs an evaluation method for the importance of nodes in power system grids, which can effectively and accurately evaluate the importance of nodes in complex power system grids. The proposed algorithm takes into account both local information and global position information of the grid topology, and integrates multiple attributes of nodes: H-index, core centrality, and structural hole characteristics, effectively compensating for the shortcomings of a single attribute. The evaluation results reflect more comprehensive information and can assess the importance of nodes more accurately.

The analysis of the invulnerability of power grids is typical and universal. Therefore, this paper applies network invulnerability to judge the evaluation performance of node importance. The comparison of numerical experimental results shows that the node importance evaluation algorithm proposed in this paper is relatively effective.

When the power grid system is attacked, the invulnerability of the grid system is related to the network topology and the operation mode of the lines. Increasing the transmission power limit of the grid system lines can reduce the degree of damage caused by cascading failures to the grid. Adjusting the grid topology and changing the operation mode of the lines can improve the invulnerability of the grid system.

In future research, we will consider the interaction effect between nodes measured by the power transmission on different lines in the process of power transmission in a power system grid, so as to improve the accuracy and practicability of node importance evaluation.

Author Contributions

Conceptualization, X.H.P.; data curation, H.D.; formal analysis, H.D.; funding auquisition, X.H.P.; investigation, X.L.C.; methodology, X.H.P.; project administration, R.H.W.; resources, L.Z.; software, H.D.; supervision, X.H.P.; writing-original draft, X.H.P.; writing-review and editing, X.H.P.. All authors have read and agreed to the published version of the manuscript.

All authors reviewed the manuscript.

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